Indices or Powers

mc-indices1-2009-1

A **power**, or an **index**, is used when we want to multiply a number by itself several times. It enables us to write a product of numbers very compactly. The plural of index is **indices**. In this leaflet we remind you of how this is done, and state a number of rules, or laws, which can be used to simplify expressions involving indices.

Powers, or indices

We write the expression

$$5 \times 5 \times 5 \times 5 \quad \text{as} \quad 5^4$$

We read this as 'five to the power four'.

Similarly

$$a \times a \times a = a^3$$

We read this as 'a to the power three' or 'a cubed'.

In the expression 5^4, the **index** is 4 and the number 5 is called the **base**. More generally, in the expression b^c, the index is c and the base is b. Your calculator will probably have a button to evaluate powers of numbers. It may be marked x^y or $x\hat{\ }y$. Check this, and then use your calculator to verify that

$$5^4 = 625 \qquad \text{and} \qquad 13^7 = 62748517$$

Exercises

1. Without using a calculator work out the value of

a) 4^3, b) 5^5, c) 2^6, d) $\left(\frac{1}{2}\right)^3$, e) $\left(\frac{2}{3}\right)^2$, f) $\left(\frac{2}{5}\right)^3$.

2. Write the following expressions more concisely by using an index.

a) $a \times a \times a \times a \times a \times a$, b) $(3ab) \times (3ab) \times (3ab)$, c) $\left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right)$.

The rules of indices

To manipulate expressions involving indices we use rules, sometimes known as the **laws of indices**. The laws should be used precisely as they are stated - do not be tempted to make up variations of your own! The three most important rules are given here:

First rule

$$a^m \times a^n = a^{m+n}$$

When expressions with the same base are multiplied, the indices are added.

Examples

(a) Using the first rule we can write

$$8^3 \times 8^4 = 8^{3+4} = 8^7$$

(b) Using the first rule we can write

$$a^4 \times a^7 = a^{4+7} = a^{11}$$

You could verify the first result by evaluating both sides separately.

Second rule

$$(a^m)^n = a^{mn}$$

Note that m and n have been multiplied to give the new index mn.

Examples

$$(3^5)^2 = 3^{5\times 2} = 3^{10} \qquad \text{and} \qquad (\mathrm{e}^x)^y = \mathrm{e}^{xy}$$

Third rule

$$\frac{a^m}{a^n} = a^{m-n} \qquad \text{or equivalently} \qquad a^m \div a^n = a^{m-n}$$

When expressions with the same base are divided, the indices are subtracted.

Examples

We can write

$$\frac{9^5}{9^3} = 9^{5-3} = 9^2 \qquad \text{and similarly} \qquad \frac{a^7}{a^4} = a^{7-4} = a^3$$

It will also be useful to note the following important results:

$$a^0 = 1, \qquad a^1 = a$$

So, any number (other than zero) raised to the power 0 is 1. This result can be obtained from the third rule by letting $m = n$.

Further, any number raised to the power 1 is itself.

Exercises

3. In each case choose an appropriate law to simplify the expression:

a) $5^3 \times 5^{13}$, b) $8^{13} \div 8^5$, c) $x^6 \times x^5$, d) $(a^3)^4$, e) $\dfrac{y^7}{y^3}$, f) $\dfrac{x^8}{x^7}$.

4. Use one of the laws to simplify, if possible, $x^8 \times y^5$.

Answers

1. a) 64, b) 3125, c) 64, d) $\frac{1}{8}$, e) $\frac{4}{9}$, f) $\frac{8}{125}$.

2. a) a^6, b) $(3ab)^3$, c) $\left(\frac{a}{b}\right)^4$.

3. a) 5^{16}, b) 8^8, c) x^{11}, d) a^{12}, e) y^4, f) $x^1 = x$.

4. This cannot be simplified because the bases are not the same.

